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LETTER TO THE EDITOR

Fluctuators and qubits: coherent quantum oscillations

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Abstract
Recent experiments observed coherent quantum oscillations for a system of Josephson phase qubits and critical-current fluctuators, which demonstrated the ability to measure two-qubit interactions. Here we propose a new scheme of manipulations with a system consisting of coupled qubits and fluctuators. In such a scheme one can, on the one hand, observe quantum coherent Rabi-like oscillations. On the other hand, the scheme permits us to distinguish responses from qubits with different eigenvalues. Also, it will be useful for testing the system and extracting its parameters.

Quantum algorithms can run exponentially faster than their classical analogues [1]. Also, the information theory in the case of quantum computation can use the advantages of quantum correction protocols. Since small (of a nanometre scale) systems are governed by the laws of quantum mechanics, nano-scale components of the hardware have to manifest quantum behaviour. These features have led researchers to imagine construction of novel types of information processors. A quantum computation has to be performed through a set of transformations, gates. A gate applies a unitary transformation to a set of qubits. The latter can be considered in the simplest case as a two-level system, which is characterized by two states ('yes' and 'no' for a qubit), and the energy difference between these states. However, such a construction of quantum computers represents a formidable scientific and technological challenge. Quantum integrated circuits have to satisfy the so-called DiVincenzo criteria [2]. First, one needs to have a scalable system in which to encode the information in qubits, that lives long enough. Second, it must be possible to set the initial state of qubits before each new computation. Third, each qubit should be isolated enough from the environment to reduce effects of incoherence. Then, one should be able to manipulate with the states of individual qubits, and the gate operation time must be shorter than the incoherence time multiplied by the maximum tolerable error rate. Finally, it must be possible to measure the final state of qubits once the computation is finished, and to obtain the output of the computation. There exist sets of universal gates from which any computation can be constructed or approximated as precisely as desired, without inventing a new gate each time. Then the problem is to build such a set of gates that act only on one or two qubits at a time. Many set-ups were proposed for the realization of a quantum computer, including, just to mention a few of them, cold trapped atoms (CTAs), nuclear magnetic resonance (NMR), Josephson junctions (JJs), and electrons in quantum dots (QDs) [3]. Some of the experimental realizations of qubits and gate operations on them are the nuclear magnetic resonance in a liquid state [4] and in solids [5], cavity quantum electron dynamics [6], CTA in optical lattices [7], molecular magnets [8], two-electron quantum dots [9], and Josephson junctions (the latter in charge, phase, and flux realizations of qubits) [10].

Recent experiments on tunnel JJ qubits detected coherent quantum oscillations, the Rabi oscillations, in a system of junction resonators [11]. Rabi oscillations are believed to be the manifestation of the entanglement in a quantum system. Entanglement is, probably, one of the key features of systems, supposed to serve as quantum computers, while it is absent in their classical counterparts [1]. The observed oscillations rapidly decay, probably due to interactions with the environment. A very interesting feature of those oscillations was observed: the decay rate and the oscillation pattern were significantly suppressed in the vicinity of certain frequencies. Such a suppression reveals the strongly dependent eigenvalues of the qubit ramped by external parameters. This phenomenon was interpreted as an influence of some fluctuators (additional two-level systems) located in the qubit environment [11]. On the other hand, the experiment [12] detected other coherent quantum oscillations, as the authors believe, different from the Rabi oscillations. Motivated by those experimental results, in this work we propose the new scheme of the realization of manipulations with qubits (gates). In fact, it is a novel approach to the already known proposals [3]. This scheme, on the one hand, satisfies many of the DiVincenzo criteria in the same way, as other proposals do [3]. On the other hand, the specific feature of our scheme permits us to measure the response of each qubit (which is characterized by its own energy distance between the states of the qubit, its eigenvalue).

Basically, the theoretical description of most of the qubits in any of the realizations [3] is related to two-level systems. Then, the description of gates is connected with unitary transformations acting on these two-level systems. One of the simplest ways to consider a two-level system is to investigate some effective quantum spin $\frac{1}{2}$, which two states are related to two states of the qubit. Then, the Hamiltonian of a model of a qubit can be written as a sum of $\mathcal{H}_{\mathrm{q}} = HS^z_{\mathrm{q}}$ and $\mathcal{H}_{\mathrm{m}}$, where the latter is the 'manipulation' part, which describes some effective ac field with the frequency ω and the magnitude h, $\mathcal{H}_{\mathrm{m}} \sim h\cos\omega t$, acting on the qubit. The two-level system of the fluctuator can be written as $\mathcal{H}_{\mathrm{f}} = ES^z_{\mathrm{f}}$. Then the distances between levels of the qubit and of the fluctuator are $2H$ and $2E$, respectively. It is natural, then, to introduce the coupling between the qubit and the fluctuator of the form $\mathcal{H}_{\mathrm{qf}} = u(S^x_{\mathrm{q}}S^x_{\mathrm{f}} + S^y_{\mathrm{q}}S^y_{\mathrm{f}})$, where $2u$ is the off-diagonal coupling constant. It was shown recently [13] that the diagonal interaction between the qubit and the fluctuator does not specifically affect the visibility of Rabi oscillations as long as the fluctuator is decoupled from the ac field. On the other hand, the presence of the diagonal coupling can contribute to the decoherence of the qubit via the random modulation of the Rabi frequency. The total Hamiltonian of the pair, consisting of the qubit and the fluctuator, has the form $\mathcal{H} = \mathcal{H}_{\mathrm{q}} + \mathcal{H}_{\mathrm{f}} + \mathcal{H}_{\mathrm{qf}} + \mathcal{H}_{\mathrm{m}}$.

Let us consider the dynamics of the system, which is described by the Hamiltonian $\mathcal{H}$. For the 'manipulation' part we propose to use the form $\mathcal{H}_{\mathrm{m}} = hS^z_{\mathrm{q}}\cos\omega t$, instead of the usually considered 'standard geometry' $\mathcal{H}_{\mathrm{m}} = hS^x_{\mathrm{q}}\cos\omega t$. The dynamics of the quantum system is described by the Liouville equation for the density matrix ρ:

$$\mathrm{i}\hbar\dot{\rho} = [\mathcal{H}, \rho], \tag{1}$$

where the dot denotes the time derivative. It is convenient to use the fermionic representation of the operators of spin projections of the qubit and the fluctuator: $S^z_{\rm q} = \frac{1}{2} - a^\dagger_{\rm q} a_{\rm q}$, $S^z_{\rm f} = \frac{1}{2} - a^\dagger_{\rm f} a_{\rm f}$, $S^+_{\rm q} = a_{\rm q}$, $S^+_{\rm f} = (1 - 2a^\dagger_{\rm q} a_{\rm q}) a_{\rm f}$, $S^-_{\rm q,f} = (S^+_{\rm q,f})^+$. Here $S^\pm_{\rm q,f} = S^x_{\rm q,f} \pm {\rm i} S^y_{\rm q,f}$, and the creation and annihilation operators $a^\dagger_{\rm q,f}$ and $a_{\rm q,f}$ satisfy standard fermionic anticommutation relations. The Hamiltonian $\mathcal{H}$ in the fermionic representation has the form

$$\mathcal{H} = -(H + h\cos\omega t) a^\dagger_{\rm q} a_{\rm q} - E a^\dagger_{\rm f} a_{\rm f} + \frac{u}{2}(a^\dagger_{\rm q} a_{\rm f} + a^\dagger_{\rm f} a_{\rm q}) + \frac{H + E + h\cos\omega t}{2}. \tag{2}$$

Then we can use the representation $j^z = \frac{1}{2}(a^\dagger_{\rm q} a_{\rm q} - a^\dagger_{\rm f} a_{\rm f})$, $j^+ = a^\dagger_{\rm q} a_{\rm f}$, $j^- = a^\dagger_{\rm f} a_{\rm q}$ (cf [14]). It is easy to check that the operators j^z and $j^\pm$ satisfy the standard commutation relations for moments. As for the value of that fictitious moment, one can see that in the subspace with the basic vectors $a^\dagger_{\rm q,f}|0\rangle$ (where $a_{\rm q,f}|0\rangle = 0$, i.e. $|0\rangle$ is the vacuum state for fermions) the value of the fictitious moment is equal to $j = \frac{1}{2}$, while for states $|0\rangle$ and $a^\dagger_{\rm q} a^\dagger_{\rm f}|0\rangle$ the value of the fictitious moment is $j = 0$. Then we can re-write equation (2) in the form

$$\mathcal{H} = -(H - E + h\cos\omega t) j^z + u j^x + \mathcal{H}_{\rm c}, \tag{3}$$

where

$$\mathcal{H}_{\rm c} = \frac{H + E + h\cos\omega t}{2}(1 - a^\dagger_{\rm q} a_{\rm q} - a^\dagger_{\rm f} a_{\rm f}). \tag{4}$$

It is easy to check that $[\mathcal{H}_{\rm c}, \mathcal{H}] = 0$. At $t = 0$, i.e. without 'manipulations', four states of the considered system of the qubit and the fluctuator have the energies 0, $H + E$, and $(1/2)[H + E \pm \sqrt{(H - E)^2 + u^2}]$.

Now let us use the unitary transformation $\rho = U_1 \rho_1 U_1^{-1}$, where

$$U_1 = \exp\left({\rm i}\frac{h}{\hbar\omega}\sin\omega t j^z\right). \tag{5}$$

after which the dynamics of the system is determined by the equation ${\rm i}\hbar\dot\rho_1 = [\mathcal{H}_1, \rho_1]$, where the effective Hamiltonian $\mathcal{H}_1$ is

$$\begin{aligned}\mathcal{H}_1 &= \mathcal{H}_{\rm c} + (E - H) j^z + \frac{u}{2}\left[j^+ \exp\left({\rm i}\frac{h}{\hbar\omega}\sin\omega t\right) + j^-\left(-{\rm i}\frac{h}{\hbar\omega}\sin\omega t\right)\right] = \mathcal{H}_{\rm c} + (E - H) j^z + \frac{u}{2} \\ &\quad\times \sum_{n=-\infty}^{\infty} J_n(h/\hbar\omega)[j^+\exp({\rm i}n\omega t) + j^-(-{\rm i}n\omega t)],\end{aligned} \tag{6}$$

where we used the standard series expansion $\exp({\rm i}z\sin\omega t) = \sum_{n=-\infty}^{\infty} J_n(z)\exp({\rm i}n\omega t)$, with $J_n(z)$ being the Bessel functions.

Following [11–13] we can suppose that $u \ll H, E, |H - E|$ and $H, E, |H - E| \sim \hbar\omega$. In this case we can consider the third term in equation (6) as a perturbation. Notice that in such a case we can use the unitary transformation, after which we have ${\rm i}\hbar\dot\rho_2 = [\mathcal{H}_2, \rho_2]$, where

$$U_2 = \exp(-{\rm i}m\omega t j^z), \tag{7}$$

and the effective Hamiltonian $\mathcal{H}_2$ is equal to

$$\begin{aligned}\mathcal{H}_2 &\approx \mathcal{H}_{\rm c} + (|H - E| - m\hbar\omega) j^z + \frac{u}{2} J_{-m}(h/\hbar\omega) \\ &\quad\times [j^+\exp(-{\rm i}m\omega t) + j^-\exp({\rm i}m\omega t)] + \cdots.\end{aligned} \tag{8}$$

Here we exactly consider only resonance terms, which are distinguished from others by the condition $|H - E| = m\hbar\omega$ (where $m = 1, 2, 3, \ldots$). For those terms there exist zero denominators in the perturbation series. Other, non-resonance, terms can be taken into account in the framework of a standard perturbation theory. It is easy to see that the population $n_q = \langle a_q^\dagger a_q \rangle$ of the qubit (and similar for the fluctuator) is an oscillating function with the frequency ω, and with the frequency of Rabi-like coherent quantum oscillations $u J_m(mh/\hbar\omega)$. For example, we obtain for the change of the difference of the occupancies, caused by the 'manipulation' in resonance

$$\delta(n_q - n_f) \equiv 2\delta\langle S_q^z \rangle \sim \frac{H}{\hbar\omega} f(T) \times \sin^2\left(\frac{u J_m(mh/|H-E|)}{2\hbar} t\right), \tag{9}$$

where T is the temperature, $f(0) = 1$, and for $T \neq 0$ we have

$$f(T) = \left.\frac{\sinh(\hbar\omega/2T)}{\cosh(\hbar\omega/2T) + \cosh[(H+E)/2T]}\right|_{\hbar\omega\approx|H-E|} \approx \frac{1}{2}\left[\tanh\frac{H}{T} - \tanh\frac{E}{T}\right].$$

The frequency of Rabi-like oscillations is determined by the magnitude of the ac field h. For $m = 1$ (i.e., if $|H - E| = \hbar\omega$) we have $J_1(h/\hbar\omega) \approx h/\hbar\omega$ for the reasonable situation of the qubit in resonance $h \ll \hbar\omega$. Hence, in this case one can observe the Rabi oscillations of the populations of the qubit and the fluctuator with the frequency $hu/\hbar^2\omega$. On the other hand, one can see that near the resonance $m\hbar\omega = |H - E|$, the ratio $mh/|H - E|$ oscillates with the change of the magnitude h according to the change of the Bessel function. When $J_m(h/\hbar\omega)$ passes zero, the frequency of the Rabi-like quantum coherent oscillations and the total response both go to zero, i.e., we have modulations of the coherent frequency, depending on the frequency of the 'manipulation' ω, and eigenvalues of the qubit H and the fluctuator E. This is reminiscent of the situation observed in experiments on Josephson tunnel junction qubits [11, 12]. Notice that beating can exist if one considers $H \sim E$, at which situation the oscillations of the qubit interfere with the oscillations of the fluctuator (which seems to be the case of [11, 13]).

To be closer to the experimental situation, we can phenomenologically introduce the relaxation to the problem. Equations of motion for the average values of the fictitious moment in the presence of relaxation can be written (cf [14])

$$\begin{aligned}
\hbar\langle \dot{j}^z \rangle &= (j_0^z - \langle j^z \rangle)\hbar\gamma_1 - \mathrm{i}C(\langle j^+ \rangle - \langle j^- \rangle), \\
\hbar\langle \dot{j}^+ \rangle &= -\hbar\gamma_2\langle j^+ \rangle + \mathrm{i}\Delta\langle j^- \rangle + 2\mathrm{i}C\langle j^z \rangle, \\
\hbar\langle \dot{j}^- \rangle &= -\hbar\gamma_2\langle j^- \rangle - \mathrm{i}\Delta\langle j^+ \rangle - 2\mathrm{i}C\langle j^z \rangle,
\end{aligned} \tag{10}$$

where $\Delta \approx |H - E| - m\hbar\omega$, $C = u J_m(h/\hbar\omega)$, and we introduced the energy relaxation and the dephasing with the characteristic rates γ_1 and γ_2, respectively. Observe that the form of equation (10) implies that we have the relaxation to the state in which $\langle j^z \rangle = j_0^z$. It is natural to suppose that this equilibrium value is determined by the Gibbs distribution with the Hamiltonian $\mathcal{H}$ taken at $h = 0$. One can use, though, the relaxation to different values of $\langle j^{z,\pm} \rangle$, e.g., to their values taken at some time, where they will be determined by h and ω also. The solution of the set of equations (10) is straightforward. One can see that for $t \ll \gamma_{1,2}^{-1}$ we obtain essentially equation (9). At $t \gg \gamma_{1,2}^{-1}$, where we can omit the left-hand sides of equation (10), the magnitude of coherent quantum Rabi-like oscillations decays as $\exp(-\gamma_{1,2}t)$, and populations of the levels of the qubit and the fluctuator oscillate in that steady-state regime with the only frequency of the 'manipulation', ω. In the intermediate regime, $t \sim \gamma_{1,2}$, we obtain the decaying coherent quantum oscillations. In figure 1 the time

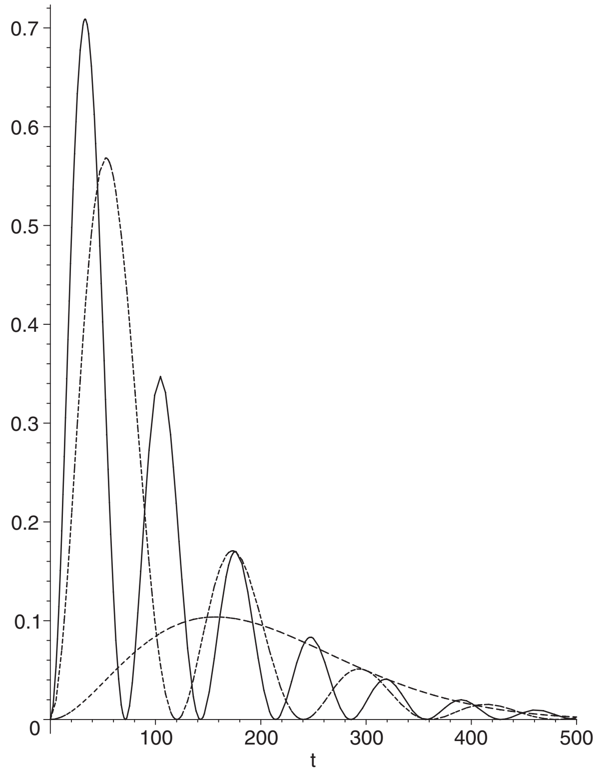

Figure 1. Time dependence of the ground state difference in populations of the qubit and the fluctuator in resonance with the 'manipulation' frequency (arbitrary units). The solid line pertains to the case $\hbar\omega = |H - E|$, i.e. $m = 1$ and $h/|H - E| = 1$; the dashed curve corresponds to the case $5\hbar\omega = |H - E|$ ($m = 5$). The dotted line describes the case $\hbar\omega = |H - E|$ ($m = 1$), but with $h/|H - E| = 0.1$.

dependence of the difference in populations of the qubit and the fluctuator are presented (the populations of the qubit and the fluctuator themselves oscillate analogously). Here we used the values $\hbar = 1$, $\gamma = 0.01$, and $u = 0.1$ (cf [11–13]). Results are presented for the resonance case $|H - E| = m\hbar\omega$ and different values of E and H (cf [12, 13]). One can clearly observe decaying coherent quantum oscillations with different frequencies, depending on the resonance conditions. In the case of large m (i.e., smaller frequencies of the 'manipulation'), the period of quantum coherent oscillations becomes larger.

One can see the similarity with the situation observed in [11, 12]. In the steady-state regime the power of the ac field, absorbed by the qubit and the fluctuator, is in resonance

$$Q \sim h^2\omega \frac{\sinh(\hbar\omega/2T)}{\cosh(\hbar\omega/2T) + \cosh[(H+E)/2T)]} \frac{\hbar\gamma_2}{(\hbar\gamma_2)^2 + \Delta^2 + 4(\gamma_2/\gamma_1)C^2}. \qquad (11)$$

Obviously, the coherent quantum oscillations, studied above, are directly related to the entanglement in the considered quantum system. The magnitudes and frequencies of those oscillations are determined not only by the eigenvalues of the qubit and the fluctuator, but also by the interaction between them.

We can consider a number of coupled qubits and oscillators. For this purpose, let us mark the operators of the qubit and the fluctuator by the index n (i.e., $S^{x,y,z}_{\mathrm{q,f}} \to S^{x,y,z}_{\mathrm{q,f},n}$). Then we can introduce the interaction between neighbouring qubits and oscillators of the form

$v\sum_n(S^x_{\mathrm{f},n}S^x_{\mathrm{q},n+1} + S^y_{\mathrm{f},n}S^y_{\mathrm{q},n+1})$. We consider such an interaction only for simplicity, to have an exactly solvable model, cf [14]. On the other hand, other forms of interactions between qubits and fluctuators can be studied in the framework of the mean-field-like theory [15]. Then, after the Jordan–Wigner transformation, followed by the Fourier transformation, we can again diagonalize the time-independent part of the Hamiltonian. Again, we can use the fact that the interactions u and v are small compared to the eigenvalues of qubits and fluctuators $u, v \ll H, E, |H - E|$, and $H, E, |H - E| \sim \hbar\omega$. Then, equation (10) describes the time dependence of the considered system of coupled qubits and fluctuators, with the changes $\langle j^{z,\pm}\rangle \to \langle j^{z,\pm}_k\rangle$, $j^z_0 \to j^z_{0,k}$, $\Delta \to \Delta_k = \sqrt{(H-E)^2 + u^2 + v^2 + 2uv\cos k} - \hbar\omega$ and $C \to C_k = |u + v\exp(\mathrm{i}k)|J_\mathrm{m}(h/\hbar\omega)$. Now it is easy to see why we support our scheme of 'manipulation'. For a set of coupled qubits and fluctuators, one expects each qubit to have its own eigenvalue (in our above consideration this corresponds to the case with different values of k). The resonance conditions $\sqrt{(H-E)^2 + u^2 + v^2 + 2uv\cos k} \approx |H - E| = m\hbar\omega$ will distinguish the number of harmonics m being in resonance modes, related to each qubit and fluctuator. For example, the difference in populations of the qubit and fluctuator with the quasi-momentum k, caused by the 'manipulation', is equal in resonance at $t \sim \gamma^{-1}$ to

$$\delta(n_{\mathrm{q},k} - n_{\mathrm{f},k}) \sim \frac{H}{\hbar\omega}\exp(-\gamma t) \times \sin^2\left(\frac{\sqrt{u^2+v^2+2uv\cos k}\,J_\mathrm{m}(mh/|H-E|)}{2\hbar}t\right) \times \frac{\sinh(\hbar\omega/2T)}{\cosh(\hbar\omega/2T) + \cosh[(H+E)/2T]} + \text{constant}, \tag{12}$$

where we used γ as the maximal value of the rates of relaxation in the system. Observing different modulations for Rabi-like coherent quantum oscillations related to different qubits (for different modes one can expect a different behaviour of zeros of frequencies) one can distinguish the responses from qubits. It turns out that in many experiments on Rabi-like coherent oscillations of qubits not only two-level systems, but also three-level systems, were used. Our scheme can be adopted for such a case, too (cf [16]).

Let us discuss the advantages of our proposal. In the proposed scheme the same quantum computation algorithms [1] can be used as in the standard scheme. Moreover, the proposed scheme does not change the subject of application (in the mentioned context it is the set of Josephson junctions), i.e., it satisfies the first and third DiVincenzo criteria as well as the previously used scheme. In our proposal the second criterion is satisfied, obviously, by applying high enough negative voltage H to qubits, which are switched to the initial state. Here we point out that in the standard scheme (with the ac field directed perpendicular to H) the operator of manipulation $\mathcal{H}_\mathrm{m} \sim h(t)S^x$ does *not* commute with $\mathcal{H}_\mathrm{q}$, and, hence, the system of qubits can be switched to the initial state formally only at $H \to -\infty$ for $h(t) \neq 0$. Using the proposed 'parallel' manipulation, one can use the fact that it is possible to measure the average value of S^z for any $h(t) \neq 0$ in the quantum mechanical sense, while it is impossible in the standard scheme. Qubits, on the other hand, stay entangled. The main advantage of our proposal can be seen when considering the fourth and fifth DiVincenzo criteria. (The gate operation times in our scheme have to be compared with the same relaxation times as in the standard scheme.) In our scheme, entangled qubits produce normal oscillation modes k. Then, because of different magnitudes of resonant Bessel functions, one can distinguish between the responses from (even coupled) qubits, and, hence, manipulate with different qubits. In the standard scheme it is much more difficult in principle; one can use different harmonics of the standard resonance to distinguish responses from different entangled qubits; however, magnitudes of higher harmonics decay very fast). Finally, our proposal will permit us to obtain the information about the coupling (entanglement) between qubits and fluctuators u more directly, comparing to the standard geometry, because resonance responses of all modes are proportional to that value.

In summary, we have proposed a new scheme of manipulation with the system of coupled qubits and fluctuators. Such a situation of coupled qubits and fluctuators was studied in recent experiments on Josephson junction qubits, which observed coherent quantum oscillations of the populations of qubits. The advantage of the proposed scheme is the possibility to distinguish the responses from different modes, due to different modulation of the Rabi-like coherent oscillations of different modes of qubits and fluctuators in resonance. We think that such a scheme can be very promising for the manipulation with qubits in the quantum computation, namely when testing systems of coupled qubits and fluctuators, and extracting the parameters of such systems, like the strength of coupling between qubits and fluctuators.

References

[1] Nelsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] DiVincenzo D P 2000 *Fortschr. Phys.* **48** 771
[3] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
Shnirman A, Schön G and Hermon Z 1997 *Phys. Rev. Lett.* **79** 2371
Gershenfeld N A and Chuang I L 1997 *Science* **275** 350
Kane B E 1998 *Nature* **393** 133
Loss D and DiVincenzo D P 1998 *Phys. Rev.* A **57** 120
[4] Vandersypen L M K, Steffen M, Breyta G, Yannouni C S, Sherwood M H and Chuang I L 2001 *Nature* **414** 883
[5] Jelezko F, Gaebel T, Popa I, Domhan M, Gruber A and Wrachtrup J 2004 *Phys. Rev. Lett.* **93** 130501
[6] Schmidt-Kaler F, Häffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* **422** 408
[7] Brennen G K, Caves C M, Jessen P S and Deutsch I H 1999 *Phys. Rev. Lett.* **82** 1060
Duan L M, Demler E and Lukin M D 2003 *Phys. Rev. Lett.* **91** 090402
[8] Ahn J, Weinacht T C and Bucksbaum P H 2000 *Science* **287** 463
[9] Zumbühl D M, Marcus C M, Hanson M P and Gossard A C 2004 *Phys. Rev. Lett.* **93** 256801
[10] Nakamura Y, Pashkin Y A and Tsai J S 1999 *Nature* **398** 786
Yu Y, Han S, Chu X, Chu S and Wang Z 2002 *Science* **296** 889
Vion D, Aassime A, Cottet A, Joyes P, Pothier H, Urbina C, Esteve D and Devoret M H 2002 *Science* **296** 886
Chiorescu I, Nakamura Y, Harmans C J P M and Mooij J E 2003 *Science* **299** 1869
[11] Simmonds R W, Lang K M, Hite D A, Nam S, Pappas D P and Martinis J M 2004 *Phys. Rev. Lett.* **93** 077003
[12] Cooper K B, Steffen M, McDermott R, Simmonds R W, Oh S, Hite D A, Pappas D P and Martinis J M 2004 *Phys. Rev. Lett.* **93** 180401
[13] Galperin Y M, Shantsev D V, Bergli J and Altshuler B L 2005 *Preprint* cond-mat/0501455 (unpublished)
[14] Kleiner V Z and Tsukernik V M 1974 *Fiz. Met. Metall.* **37** 231 (in Russian)
Zvyagin A A 1988 *Fiz. Nizk. Temp.* **14** 661 (in Russian)
Zvyagin A A 1988 *Sov. J. Low Temp. Phys.* **14** 366 (Engl. Transl.)
[15] Zvyagin A A 1990 *Fiz. Nizk. Temp.* **16** 80 (in Russian)
Zvyagin A A 1990 *Sov. J. Low Temp. Phys.* **16** 41 (Engl. Transl.)
[16] Zvyagin A A, Frishman A M and Tsukernik V M 1983 *Fiz. Nizk. Temp.* **9** 308 (in Russian)
Zvyagin A A, Frishman A M and Tsukernik V M 1983 *Sov. J. Low Temp. Phys.* **9** 155 (Engl. Transl.)